

# DIAGONAL CIRCUIT IDENTITY TESTING AND LOWER BOUNDS

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## Abstract of Dagstuhl Talk

In this talk we give a deterministic polynomial time algorithm for testing whether a *diagonal* depth-3 circuit  $C(x_1, \dots, x_n)$  (i.e.  $C$  is a sum of powers of linear functions) is zero. We also prove an exponential lower bound showing that such a circuit will compute determinant or permanent only if there are exponentially many linear functions. Our techniques generalize to the following results:

1. Suppose we are given a depth-3 circuit of the form:

$$C(x_1, \dots, x_n) := \sum_{i=1}^k \ell_{i,1}^{e_{i,1}} \cdots \ell_{i,s}^{e_{i,s}}$$

where,  $\ell_{i,j}$ 's are linear functions living in  $\mathbb{F}[x_1, \dots, x_n]$ . We can test whether  $C$  is zero in deterministic time  $\text{poly}(nk, \max\{(1 + e_{i,1}) \cdots (1 + e_{i,s}) \mid 1 \leq i \leq k\})$ . This immediately gives a deterministic  $\text{poly}(nk2^d)$  time identity test for general depth-3 circuits of degree  $d$ .

2. We prove that if the above circuit  $C(x_1, \dots, x_n)$  with a “small”  $s = o\left(\frac{m}{\log m}\right)$  computes the determinant (or permanent) of an  $m \times m$  matrix then  $k = 2^{\Omega(m)}$ .

Our results work for all fields  $\mathbb{F}$ . (Previous exponential lower bounds for depth-3 only work for nonzero characteristic.)